57th MEETING OF THE
SOCIETY FOR NATURAL PHILOSOPHY

Complex Material Structures, Natural and Architected

Book of abstracts

edited by Roberta Zarcone

GSA Laboratoire Géométrie Structure Architecture
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Complex Material Structures, Natural and Architected

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Contents

Presentation

The Society for Natural Philosophy (SNP), founded in 1963 by Clifford Truesdell, has a specific focus on the unity of the mathematical and physical sciences. It seeks to recognise and promote high quality work.

The 57th Society for Natural Philosophy meeting is organised by three laboratories from three institutions of PSL University: the Ecole nationale superieure d'architecture Paris-Malaquais (ENSA Paris-Malaquais), the Ecole supérieure de physique et de chimie industrielles de la ville de Paris (ESPCI) and the Mines Paris.

The topic of this edition of the meeting is 'Complex Material Structures, Natural and Architectural'. It aims at strengthening the links between the disciplines taught in these institutions, in particular materials science and architecture, physics, mathematics and engineering.

The meeting is dedicated to the memory of Gianfranco Capriz (1925-2022) and Maurizio Brocato (1962-2023).

This book collects contributions on 5 different topics:

- Session 1: Generalised continua (chaired by Paolo Podio-Guidugli). Session dedicated to the memory of Gianfranco Capriz.
- Session 2: Complex materials (chaired by Samuel Forest).
- Session 3: Architectured materials (chaired by Roberta Zarcone). Session dedicated to the memory of Maurizio Brocato.
- Session 4: Mechanics of active and frustrated systems (chaired by Roberto Paroni).
- Session 5: Amorphous, glassy and granular systems (chaired by Lev Truskinovsky).

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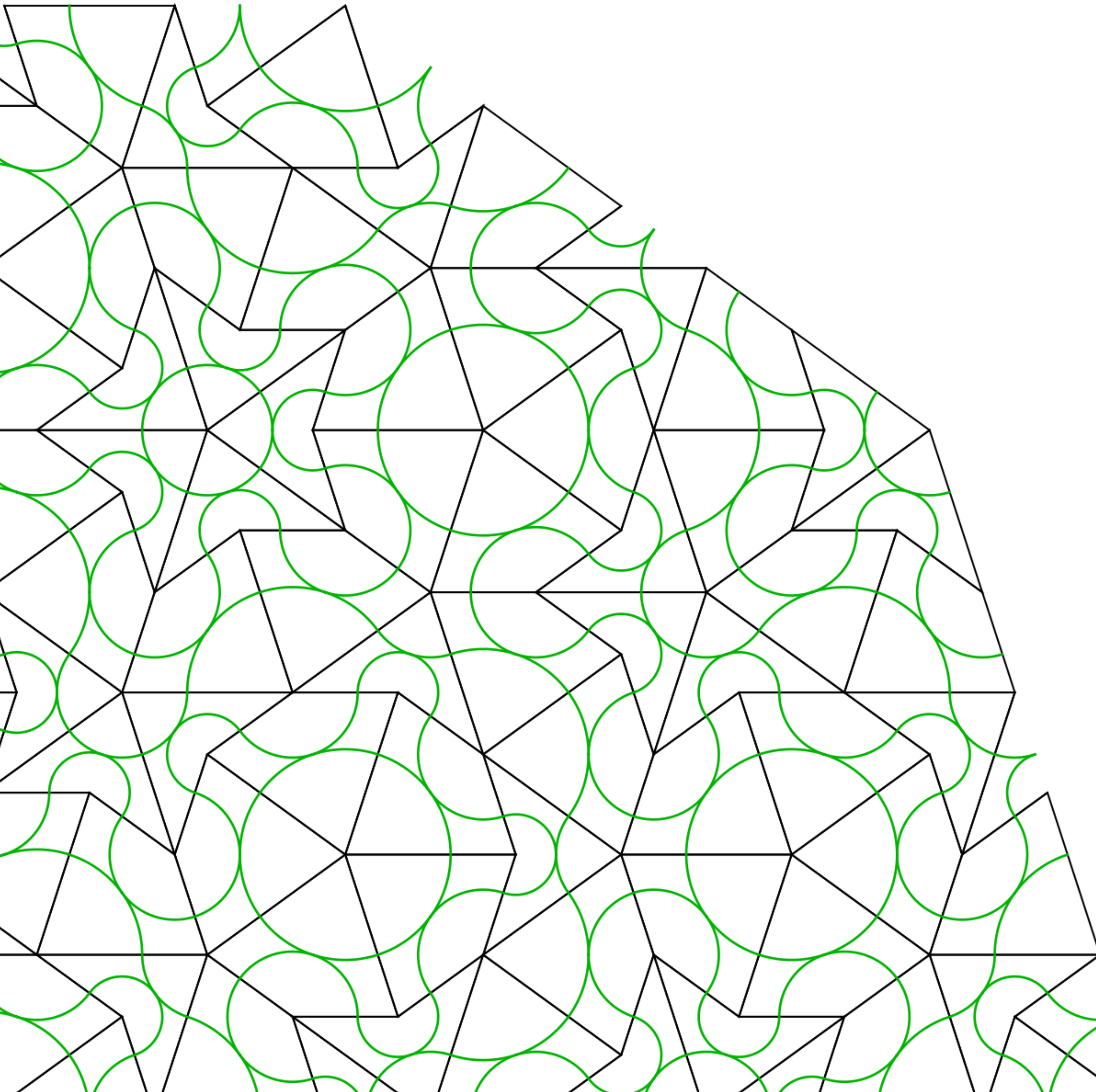
Session 1: Generalised continua



Keynote lecture

The entropy of thermodynamic systems out of equilibrium

Joe D. Goddard

Department of Mechanical and Aerospace Engineering, University of California, San Diego

Most existing mathematical theories of the second law of thermodynamics, as e.g. expressed by the Clausius-Duhem inequality, take entropy and temperature as primitives, without regard for observability or computability in non-equilibrium states. A detailed critique is presented in the recent paper by the present author, "The second law of thermodynamics asvariation on a theme of Carathéodory." [1]

In the above-cited paper it is postulated that *the difference* $\eta_{eq} - \eta$ *between (maximum) equilibrium entropy* η_{eq} *at a given given internal energy and configuration and the non equilibrium entropy* η *is given by* $\eta_{eq} - \eta = w_m/\theta$ where w_m is the maximum work that could be obtained by conversion of heat from a reservoir at the equilibrium temperature θ. This quantity is given as a functional of the history-dependent forces defining the cotangent space of the configuration space which leads to a variational problem involving a functional offorces and internal energy.

The present work provides a slightly amended version of functional to be maximized and a discussion of the possible relation to various efforts to extract a Helmholtz free energy from linear viscoelasticity. A brief discussion is also given of some necessary modifications or reinterpretations of the conventional Clausius-Duhem inequality.

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Round-table speakers

Cauchy's stress theorem *more atomistico demonstratum*

Antonio Di Carlo

Two hundred years ago, Cauchy stated and proved two different versions of his stress theorem: one for continuous media (*corps considérés commes des masses continues*) and one for systems of interacting point-like particles (*système[s] de points matériels sollicités par des forces d'attraction ou de répulsion mutuelle*). The first version is widely known, having been somehow repeated (in a more or less satisfactory manner) on any primer in continuum mechanics and/or its diverse applications. After the 1950's, keen efforts have been placed on modernizing, refining, and extending its original formulation. The second version encountered a different fate. It never entered the syllabi of either continuum or atomistic mechanics, being left aside as a marginal curiosity for historians and philosophers of science, irrelevant to practicing mechanicians.

In this talk, I will sketch an up-to-date reformulation of the atomistic version of Cauchy's stress theorem, founded on the current understanding of the macroscopic and the microscopic mechanics, both of which have undergone tremendous changes - largely unconnected with each other - over the intervening two centuries.

The smallest chunks of space on which it makes sense to match a continuum picture with an underlying atomistic description are convex polyhedral cells whose size is mesoscopic, i. e., at least a few orders of magnitude larger than the effective range of short-ranged particle interactions - which is of the order of 1 nm. The whole apparatus of localization arguments exploited in continuum mechanics has to be adapted accordingly. Here, arguments based on length-scale separation are preliminarily employed to extract a macroscopic velocity field, approximately affine on all mesoscopic space cell, from the microstate (i. e., the set of position/velocitypairs) of the particle system. Then, similar arguments are used to justify defining a vector-valued surface density - the traction vector field - on the boundary of each mesoscopic space cell, which is (i) approximately affine on each cell face, and (ii) equipowerful, with respect to all affine velocity field, to the force distribution exerted by the particles outside the cell on the particles in the cell. In conclusion, the analogues of Cauchy's postulate, fundamental lemma, and fundamental theorem are proved from Newtonian particle mechanics, under the hypothesis of a large enough scale separation between the significant wavelengths of the macroscopic stress field and the range of the underlying microscopic interactions.

On proto-Galilean dynamics

Reuven Segev[1] and Marcelo Epstein[2]
[1]Ben-Gurion University; [2]University of Calgary

The Aristotelian structure of spacetime $\mathcal{E}$ is that of a Cartesian product $\mathbb{R} \times \mathbb{E}^3$ where $\mathbb{R}$ is isomorphic to the time axis and $\mathbb{E}^3$ is a fixed three-dimensional Euclidean space. Thus, for a given event, both time and location are absolute. The Galilean structure of spacetime no longer assumes an absolute space, yet, absolute time is an essential feature of classical mechanics. Thus, Galilean spacetime has a structure of a fiber bundle

$$\vartheta : \mathcal{E} \to \mathbb{R} \tag{1}$$

assigning to each event the time it occurred. The typical fiber of the fiber bundle is $\mathbb{E}^3$, but there is no natural correspondence between the fiber $\mathcal{E}_t$ and $\mathcal{E}_{t'}$ for distinct instances t and t'. Newtonian mechanics assumes that $\mathcal{E}$ also has an affine structure, giving rise to the notion of an inertial frame.

We use the term *proto-Galilean* spacetime for the case where $\mathcal{E}$ has a fiber bundle structure as in (1), so that time is absolute; however, the typical fiber of the bundle is a general manifold. In [2], we propose a formulation of the kinematics and dynamics of a particle and a continuous body in a proto-Galilean spacetime.

In [1], a formulation of kinematics and stress theory is presented where configurations of material bodies are viewed as sections of a fiber bundle

$$\pi : \mathcal{Y} \to \mathcal{X} \tag{2}$$

(see also [3]). The base manifold $\mathcal{X}$ is interpreted as the body manifold of continuum mechanics, and the typical fiber of the fiber bundle is interpreted as the space manifold. Thus, each material point, $x \in \mathcal{X}$, "sees" another version of the space manifold.

This general setting applies naturally to the mechanics of a particle in a proto-Galilean spacetime, where a motion is a section

$$\kappa : \mathbb{R} \to \mathcal{E} \tag{3}$$

In other words, the motion of a particle is viewed as a configuration of the time axis in spacetime. A virtual displacement field in the continuum mechanics picture is a variation of the motion of the particle, while the tangent mapping to the section—the generalization of the deformation gradient—represents the velocity of the particle. The stress represents the momentum of the particle and the relation between the momentum and the velocity—specifying the inertial properties of the particle—is given by the constitutive relation determining the stress in terms of the tangent to the deformation. Finally, the equation of motion is the proto-Galilean dynamics analog of the equilibrium equation.

A similar, albeit more complicated, setting applies to the mechanics of a continuous body in a proto-Galilean spacetime.

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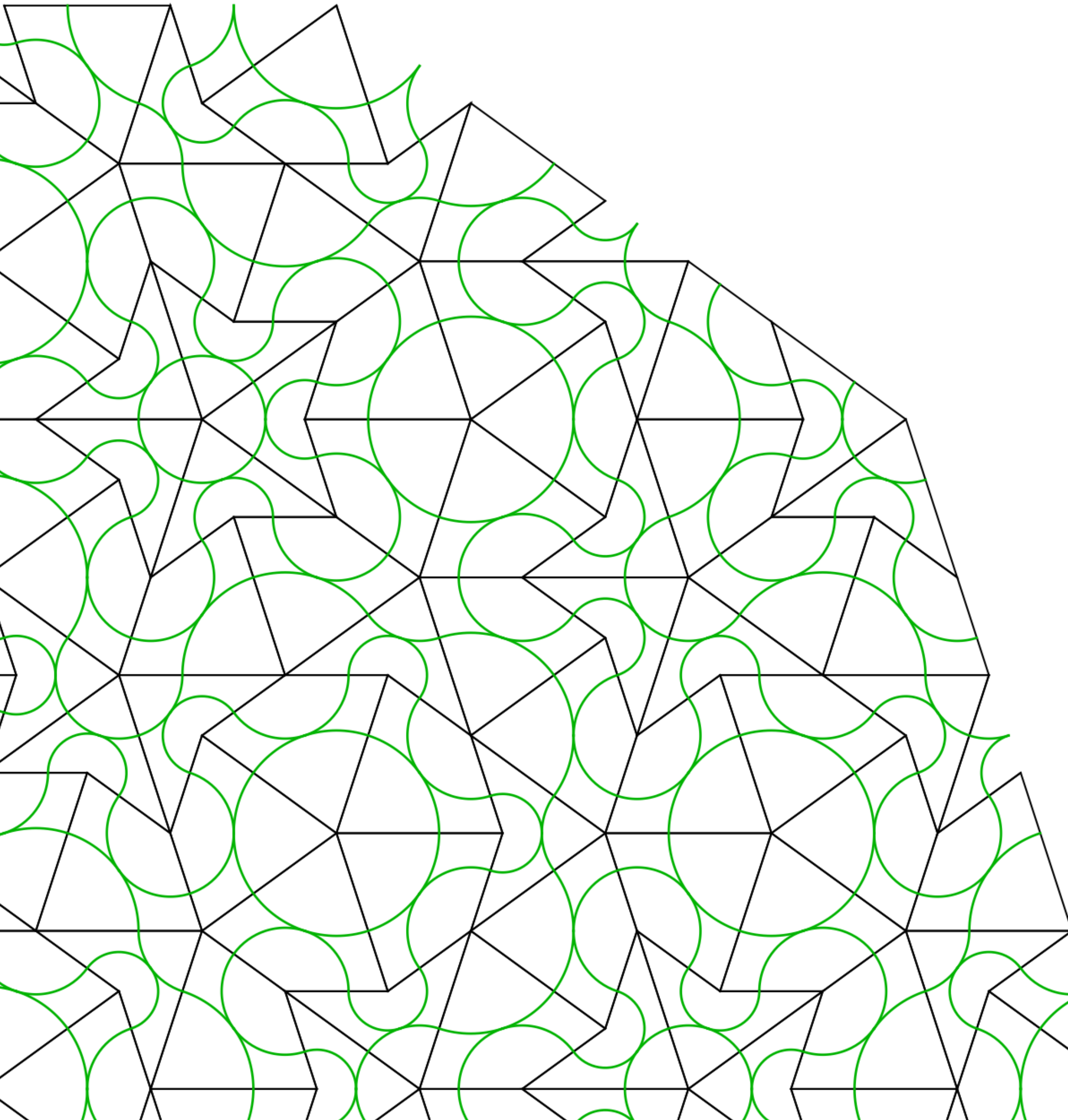
Session 2: Complex materials



Keynote lecture

Guiding Stress : from pentamodes to cable webs to masonry structures

Graeme W Milton
University of Utah

Pentamode materials are a class of materials that are useful for guiding stress. In particular, they have been proposed for acoustic cloaking by guiding stress around objects, and have been physically constructed. A key feature of pentamode materials is that each vertex in the material is the junction of 4 double cone elements.

Thus the tension in one element determines the tension in the other elements, and by extension uniquely determines the stress in the entire metamaterial. Here we show how this key feature can be extended to discrete wire networks, supporting forces at the terminal nodes and which may have internal nodes where no forces are applied. In usual wire or cable networks, such as in a bridge or bicycle wheel, one distributes the forces by adjusting the tension in the wires.

Here our discrete networks provide an alternative way of distributing the forces through the geometry of the network. In particular the network can be chosen so it is uniloadable, i.e. supports only one set of forces at the terminal nodes. Such uniloadable networks provide the natural generalization of pentamode materials to discrete networks. We extend such a problem to compression-only 'strut nets' subjected to fixed and variable nodal loads.

These systems provide discrete element models of masonry bodies, which lie inside the polygon/polyhedron with vertices at the points of application of the given forces ('underlying masonry structures'). In particular, we solve the two-dimensional problem where one wants the strut net to avoid a given set of obstacles, and also allow some of the forces to be reactive ones.

This is joint work with Ada Amendola, Guy Bouchitté, Andrej Cherkaev, Antonio Fortunato, Fernando Fraternali, Ornella Mattei, and Pierre Seppecher.

Round-table speakers

Probing elastic binodal by nucleation

Yury Grabovsky
Temple University, Philadelphia,USA

Microstructural complexity observed during martensitic phase transitions in solids could be explained by a failure of the underlying physically informed energies to be rank-one convex.

The resulting microstructure is determined by phase boundaries, whose stability is intimately linked to stability of homogeneous configurations in the hard device (affine Dirichlet boundary conditions).

The central object of study is the *binodal* - a hypersurface in the phase space separating stable and unstable homogeneous configurations.

In this presentation, reporting on the joint work with Lev Truskinovsky, I will describe the tools we have developed for probing the binodal: the jump set, progressively more sophisticated tests of its stability,and nucleation tests.

I will illustrate the strengths of our methods on a previously intractable example of two-well Hadamard materials, where we have been able to gain a surprisingly detailed insight into the structure of their binodal.

Variational formulation for hierarchical structured deformations

Marco Morandotti
Politecnico di Torino, Italy

The theory of structured deformations proposed by Del Piero and Owen [3] provides a mathematical framework to capture the effects at the macroscopic level of geometrical changes at submacroscopic levels.

The availability of this framework leads naturally to the enrichment of the energies and force systems that underlie variational and field-theoretic descriptions of important physical phenomena without having to commit at the outset toany of the existing prototypical mechanical theories, such as elasticity or plasticity.

A variational formulation, amenable to use the techniques of the calculus of variations, of this theory has been proposed by Choksi and Fonseca [2] in the context of special functions of bounded variation. In the abovementioned works, a single submacroscopic level has been considered, even though many natural and man-made physical systems have a rich enough structure to allow one to identify many meaningful submacroscopic levels.

In this direction, Deseri andOwen [4] proposed an enrichment of the theory of structured deformations to consider many submacroscopic levels. In this talk we will present the variational formulation of such a theory [1], showing how to assign an energy to a hierarchical structured deformation by means of the relaxation of an initial energy featuring both volume term and interfacial terms.

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Measure-valued plastic factors and related energy minimization in SBV spaces

Paolo Maria Mariano
DICEA, University of Florence, Italy

Consider a single crystal occupying a domain Ω, a fit region in $\mathbb{R}^3$, endowed with piecewise Lipschitz boundary. When a single slip plane S activates along a deformation $\varphi : \Omega \to \mathbb{R}^3$, so that φ jumps across S, per se φ is a SBV map with distributional derivative $D\varphi = \nabla\varphi\,\mathcal{L}^3 + b \otimes \nu\,\mathcal{H}^2 \llcorner S$, where $b \in \mathbb{R}^3$ is the pertinent Burgers vector, $\mathcal{L}^3$ the $3D$ Lebesgue measure, and $\mathcal{H}^2$ the $2D$ Hausdorff's one. By assumption φ satisfies the orientation preserving condition outside the slip plane, that is $\det \nabla\varphi > 0$ a.e. in Ω. With the standard notation $F := D\varphi$, we thus get the multiplicative decomposition of the "deformation gradient", namely

$$F = F^e F^p \; ,$$

into so-called "elastic" (F^e) and "plastic" (F^p) factors, given respectively by

$$F^e = \nabla\varphi \, , \qquad F^p = I\,\mathcal{L}^3 + (\nabla\varphi)^{-1}(b \otimes \nu)\,\mathcal{H}^2 \llcorner S \; ,$$

where I is the unit tensor represented by a 3×3 identity matrix. Also, since $\mathrm{curl} D\varphi = 0$, we compute $\mathrm{curl}(\nabla\varphi\,\mathcal{L}^3) = -b \otimes \tau \mathcal{H}^1 \llcorner \Gamma$, where $\Gamma = \partial S$ is oriented by the vector τ.

Thus, the plastic factor F^p appears naturally as a measure. Taking into account this aspect, for rearrangements of matter dominated by slip mechanisms occurring over a more general slip system indicated once again by S and considered to be a $2D$ rectifiable set with boundary Γ, we accept once again the multiplicative decomposition $F = F^e F^p$, where F^p is now a $\mathbb{R}^{3\times 3}$-valued bounded measure in Ω, which decomposes as

$$F^p = a(x) I \mathcal{L}^3 + \hat{F}(\bar{S}, \overline{\Gamma})$$

with $a(x)$ a measurable function in Ω satisfying $C^{-1} \leq a(x) \leq C$ for all $x \in \Omega$ and some given real constant $C > 1$; the presence of $a(x)$ allows us to account for plastic volume variations. $\hat{F}(\bar{S}, \overline{\Gamma})$ is a tensor-valued measure, supported over a $2D$-rectifiable set ($\bar{S}$ and $\overline{\Gamma}$ are currents associated with S and Γ), while $\mathrm{curl} F^p = \mathrm{curl} \hat{F}(\bar{S}, \overline{\Gamma})$ is a measure on a $1D$-rectifiable set.

Consider an energy given by

$$\widetilde{\mathcal{F}}(\varphi) := \int_\Omega \tilde{e}(x, F^e)\,\mathrm{d}x + |\mathrm{curl} F^p|(\Omega) + \mathbf{S}(\overline{\Gamma}) \ ,$$

where $\tilde{e}$ is a polyconvex function of $F^e \in L^1(\Omega, \mathbb{R}^{3\times 3}; |F^p|)$, with $|\mathrm{curl} F^p|$ the total variation of $\mathrm{curl} F^p$, an energetic contribution of the Burgers tensor associated with slips, and $\mathbf{S}(\overline{\Gamma})$ a constant-density line energy.

With reference to $\widetilde{\mathcal{F}}(\varphi)$, in my talk I will discuss an existence theorem of minimizers for that energy, a result obtained with Domenico Mucci [1], and related issues concerning the definition of dissipation distances and the extension of $\widetilde{\mathcal{F}}$ to quasi-periodic crystals where a line energy for meta-dislocations appears to be natural.

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The isotropic-nematic transition on a spherical surface

Gaetano Napoli[1] and Luigi Vergori[2]
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Nematic order on curved surfaces represents an intriguing field to explore, motivated by mathematical elegance [1] and applications in soft condensed matter [2, 3, 4]. In nematic liquid crystals, temperature determines the phase transition from the isotropic state, in which there is no directional order, to the nematic state. The uniform isotropic state is only possible on certain types of surfaces, including spherical ones. On a sphere, the presence of nematic order clashes with topology of the surface. In fact, according to the Poincaré-Hopf theorem, a field of tangential directions on a sphere cannot be defined everywhere. Consequently, nematic order cannot arise uniformly on the surface; on the contrary, there will always be points at which liquid crystal melts.

We examine the problem within the framework of Landau-de Gennes theory of nematic liquid crystals [6]. In this framework, the problem can be reformulated as a bifurcation problem, where the nematic ordering bifurcates from the isotropic state as the temperature decreases [7]. By means of a weakly nonlinear analysis we compute the critical temperature, determine analytically the nematic textures and, hence, the position of the melting points and their charges at the isotropic/nematic transition. We illustrate the solutions that we have obtained, and discuss their stability.

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Principle of Virtual Work as Foundational Framework for Metamaterial Invention and Rational Design

Francesco dell'Isola

Memocs and Diceaa Università dell'Aquila; CNRS Fellow-Ambassadeur ENS Paris Saclay; Faculty of Mechanical Engineering Warsaw University of Technology

Novel theories and methods are needed for the invention of innovative and exotic metamaterial. Albeit up to now they were looked for "andomly", simply based on intuition and so-called physical sense, we claim that their design must be based on rational methods.

In fact, the current most diffused practice of mechanical analyses, based upon moribund classical theories and experimental trial-error campaigns, is blocked and caught in an inescapable vortex and illusion of inductive reasoning. The needed research paradigm to be used is at the same time very old and very modern: it is that one in which the formulation of theoretical concepts precede their experimental validation. In the absence of theoretical understanding, the design experiments and collection of experimental evidence will remain unavoidably circumscribed and any research is vain.

In fact, history of science can provide us guidance in the search for the needed powerful tools required for invention and discovery. The principle of virtual work seems to provide the only necessary framework for development of theories that can lead to novel metamaterials, as it was the unifying principle which allowed the French-Italian School, headed by D'Alembert, Lagrange and Gabrio Piola, to found modern continuum mechanics.

Based upon this framework we have conceived a metamaterial synthesis schema that exploits micro-macro identification traceable to the early days of the formulation of continuum theories for deformable solids. The schema is illustrated with application to metamaterials with pantographic and granular motifs based upon higher-gradient and higher-order theories.

Reaction-diffusion-drift equations for scintillators. From multi-scale mechanics to Gradient Flows and Wasserstein measures

Fabrizio Daví
Dipartimento di Ingegneria Civile Edile e Architettura, Università Politecnica delle Marche, Italy

When scintillating crystals are hit by an ionizing radiation (γ-*rays*) charged particles from the valence bands jump to the conduction band. These particles generates a particle shower and then recombines, some of them giving back the initial energy as photons in the visible range. For this wavelength-shifter behaviour they are used as radiation sensors in High-Energy Physics, Medical Imaging, Astrophysics and Security devices [1]. The whole phenomena is a multi-scale one: the generation of charged particles is a microscopic phenomena, their evolution and recombination take place at the mesoscopic scale and then the light propagation within the crystal obeys the law of anisotropic optics and photoelasticity.

In [2], we obtained a continuum model for the evolution and recombination of charge carriers in inorganic scintillators by modelling the crystal as a continua with microstructure [3] in the isothermal and non-deformable case the resulting evolution equation, which follows from the microforce balance, are a Reaction-Diffusion-Drift boundary value problem for the charge carriers n and the electric potential φ:

$$\begin{gathered} div(\mathbb{D}[grad\,n] + \mathbb{M}\mathbb{N}[q \otimes grad\,\varphi] - r(n) = \dot{n}\,, \quad -\epsilon\Delta\varphi = eq\cdot n\,, \text{ in } \Omega\,, \\ \mathbb{D}[grad\,n]\cdot m + \mathbb{M}\mathbb{N}q(grad\,\varphi\cdot m) = 0\,, \qquad [\![grad\,\varphi]\!]\cdot m = 0\,, \text{ on } \partial\Omega\,, \end{gathered} \tag{4}$$

in (4) n is the k-dimensional array of charge carrier densities, $q \in \mathbb{Z}^k$ is the charge vector, $\mathbb{D}$ and $\mathbb{M}$ are the $k\times k$ Diffusivity and Mobility matrices, $\mathbb{N} = diag\{n_1\,, \dots\,, n_k\}$, $r(n)$ is the recombination term, ϵ the crystal permittivity and m the outward uit normal to $\partial\Omega$.

Equations $(4)_1$ are the gradient-flow of a dual dissipation potential $\Psi(g\,, grad\,g)$ [4]:

$$\dot{n} = -D\Psi(g\,, grad\,g)\,, \quad g = eq\varphi + \theta k_B\mathfrak{F}'(n)\,, \tag{5}$$

with $\mathfrak{F}(n)$ an entropy functional and k_B the Boltzmann constant.

Here we shall deal first with some constitutive issues, namely different choices for the dissipation potential which lead to different formulations of $(4)_1$: then we shall show that the gradient flows are of the Wasserstein type and briefly discuss the associated mathematical problems .

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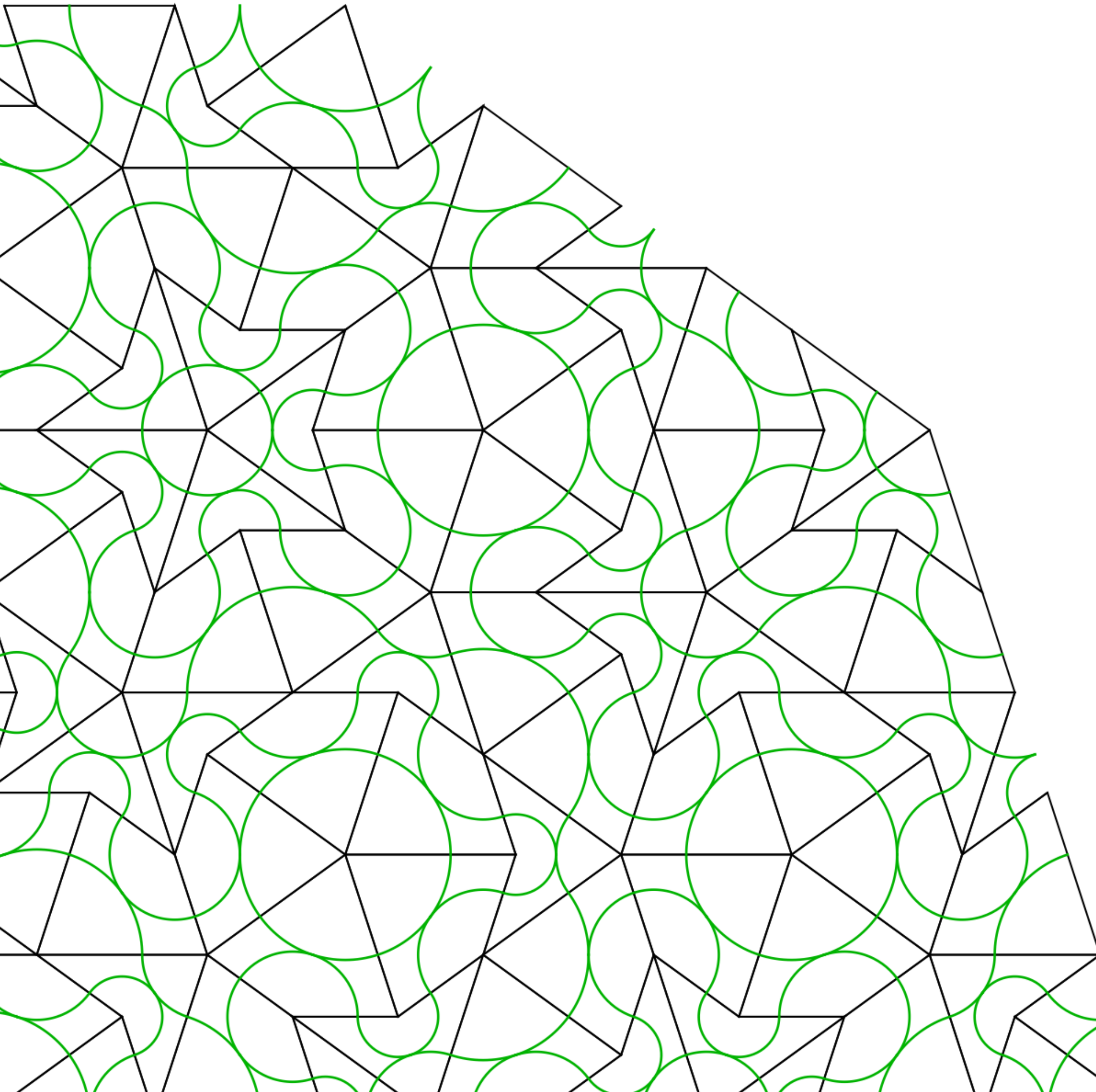
Session 3: Architectured materials



Keynote lecture

Continuum models for masonry

Maurizio Angelillo

Dipartimento di Ingegneria Civile, Università degli studi di Salerno, Italy

Most Modern Engineers have lost the ability to comprehend the nature of masonry equilibrium, since traditional masonry construction is not taught anymore in our Universities.

As a consequence of this insensitivity and lack of confidence it often happens that a typical and safe cracking pattern is interpreted as damage, then calling for extreme and heavy retrofitting interventions, if this is not an excuse to tear the building down.

The unilateral model of Heyman, translated into continuum mechanics restrictions, captures the very essence of masonry behaviour, giving a sensible answer to the masonry challenge. The equilibrium BVP for this material can be formulated variationally as the minimization of a convex functional and admit a primal and a dual formulation in terms of Potential and Complementary Energy, for which the competing strain and stress functions can be singular, namely line Dirac deltas Using Continuum Mechanics to model DISCONTINUITIES AND FORCE CHAINS, by extending the concept of strain to model cracks we see in masonry, and that of stress to reproduce force chains and force networks that may form inside them, may appear a little extreme but it turns out to be exceptionally useful.

This variational frame set the stage for the creation of new numerical methods for the analysis of masonry structures with the final goal of providing Architects and Engineers with accessible techniques and computer programs to structurally assess unreinforced masonry systems. Some applications of these new methods are shown.

Round-table speakers

Architecture and Mechanics in the proto-encyclopedic knowledge of *Mathesis Universalis*

Antonella Mastrorilli

Laboratoire LACTH, Ecole Nationale Supérieure d'Architecture et de Paysage de Lille (ENSAPL), Université de Lille, France

This talk is the result of a work begun with Maurizio Brocato at a time when Edoardo Benvenuto, Professor at the Faculty of Architecture in Genoa, promoted a series of colloquiums entitled "Between Mechanics and Architecture".

In this work, we thought it would be interesting to look at constructive thought in a field of knowledge in which the systematic and encyclopaedic structure came up against the innovative scope of seventeenth-century science. We are talking about the cultural and literary project of the Mathesis Universalis and the role it played in the relationship between science and architecture.

These were treatises on mathematical roots, language and forms, comprising several disciplines, both theoretical and applied, with the ambition of contain all human knowledge and unite it under the universal language of mathematics. Aristotelian reverberations, cartesian metaphysics, newtonian cosmology and german logic, merge with the traditional principles of construction practice and constructive precepts, in the very rigid mesh of mathematical method and language. In particular, this work analyses the Mathesis of François-Millet Dechales and Christhian Wolff, drawing parallels between the treatises on architecture and mechanics in order to identify their explicit or potential relationships.

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Resistance of flat vaults with respect to their stereotomy

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This talk presents a part of the works developed in the frame of the Phd of Mathias Fantin under the direction of Maurizio Brocato in association with Thierry Ciblac.

This research aims at a better understanding of the influence of stereotomy on the stability of stone-cut flat vaults. The mechanical study of flat arches can use elastoplastic approaches considering behaviour law. Such a study is developed in [1]. On the contrary, yield design allows a mechanical study using only resistance criteria, without consideration of behaviour law. The stability of a structure is expressed through the potentially admissible loadings domains. We use the thrust network method [2] and its extensions considering joints [3] and refined networks [4], to explore equilibrium solutions that are necessary to compute the admissible loadings domains of yield design.

The potentially admissible loadings are determined considering friction criterion and limited resistance strength at joints according to the resultant forces acting on them. The refinements of the networks using additional branches with specific properties widen the set of equilibrium solutions that can be computed and allow the exploration of complex stereotomy. The paper first presents flat vault stereotomy in architecture. Three categories of flat arches are considered: unidirectional and bidirectionnal flat vaults and Abeille's flat vaults. In this study, the flat arch cut like a circular arch is called unidirectionnal flat vault, and the flat arch cut like a pavillon vault is called bidirectionnal flat vault. Abeille's flat vaults are named after their inventor, Joseph Abeille. They are constituted of identical voussoirs spanning in two orthogonal directions.

The voussoirs are polyhedrons whose vertical sections are isosceles trapezium. The intrados forms a continuous ceiling, and the extrados forms a floor with pyramidal gaps, and they can be inversed. In the second part, we present admissible thrust networks and potentially admissible loadings domains for these three kinds of flat vaults. Unrefined networks provides stable equilibrium solutions for unidirectionnal and bidirectionnal flat vaults that can be used to compute admissible loadings domains. With refined networks, the domains computed are wider. In the case of Abeille's flat vault, the particular stereotomy of the blocks implies that the unrefined networks cannot give equilibrium solution compatible with the friction criterion. The potentially admissible loadings domain is constructed with the refined network solutions.

The particular shape of the refined network shows in a dramatic way that thrust networks going outside the limits of the masonry can be related to stable equilibrium states with respect to the joints.

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Structural stone research of Maurizio Brocato

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For more than 10 years, Maurizio Brocato's work on cut stone structures has continued to push further the limits of designs for this type of construction. In his approach, structural mechanics problematics have always been closely linked to architectural choices.

Aesthetic decisions are indeed more often the result of mechanical principles, rather than the opposite. In the case of these novel cut stone structures (interlocking domes, flat vaults, interlocking walls, interlocking saddle vaults, minimal interlocking geodesic vaults, stone beams), the approaches concerning their stability are essentially based on the local description of the joint interface between two blocks. This elementary description associated with the heavy material strength gives the overall structure all its resistance capacities and even brings out fully-fledged behaviors on the whole structure scale.

The great mastery of numerical methods with nonlinear finite elements has made it possible to better understand and precisely anticipate the mechanical nature of these objects. This expertise has also been put to good use in several contexts in recent years, and in particular within the Scientific Project for the reconstruction of Notre-Dame de Paris cathedral.

These specific modeling techniques, used and improved for years, is now used to accurately describe the mechanical behavior of the cathedral vaults and to assist the engineers and heritage architects in reconstruction decisions.

Breaking the mould: New materiality in fibre composites (FRP), as architectured matter

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Fibre reinforced polymers (FRP) offer unique opportunities for architecture, thanks to their exceptional properties. However, their mould-based forming processes stand in contrast to contemporary architectural practices, aiming for high variation and surface articulation. In search of alternative shaping processes that replace the reliance on moulds, a new FRP materiality is developed, FMFRP (fabric materiality FRP). Enhancing the textile qualities of the fibre constituent of the composite material, different material systems were developed. Formed by non-standard processes in contrast to industry's best-practices, these systems suggest alternative material architectures, that open-up new architectural qualities. Released from the necessity of moulds, design principles of biological composites can be integrated into synthetic FRP shaping processes, new aesthetic and performative qualities appear and resilient properties show. The suggested talk will review material systems developed over recent years through three projects: LifeObject, L_FMFRP, The Swirl.

In a *hierarchical material construction*, from fibre to an interlaced structure of knitted tubes, the Life Object installation demonstrates the lesson drawn from the bird's nest. 'FRP twigs' were shaped by *self-organisation* of knit fabric sleeves under stretching; their interlacing and bending constructed a stable volume with internal configuration of entangled materials. In this intricate active bending system, the limit between matter and structure is blurred and the single knit tube should be regarded as a low level of a hierarchical material construction. Just as it is typical of textile products as well as to biological composites. Internal intricacy is similarly present in L_FMFRP project, a thick and airy panel with internal undulating structure, obtained by layered pleated sheets. The gathering pleat is a simple *low energy* local contraction of the sheet material that generates substantial displacement and a general complex curved geometry. Pleating pattern and layering process determine the effective properties of the porous architectured material. When examined under compression, it exhibited an exceptional capacity to withstand large quasi-elastic deformation upon loading, with near-full recovery of initial state. Repeated cyclic loading revealed hysteretic behaviour, unsensitive to relative morphological variations between the different samples. The consistent hysteretic behaviour therefore appears as a property which is typical of the architectured material related its general configuration (pleating pattern and layering), with high level of resilience [developed with Prof. Maurizio Brocato]. Common to both systems is their shaping by the application of minimal external force over the material. The Swirl presents yet another approach, where shaping is the outcome of differential internal stresses within the material itself, as it is the case for shaping processes of plants, flower and leaves. Frustrated composites enhance the differential shrinkage between fibres and polymer matrix upon heating/cooling in the curing process, to generate internal stresses that result incomplex 3D shape. Dictated by its fibre patterns and asymmetry in section, upon its rapid cooling the flat sheet transforms into a surface of predicted complex geometry. The Swirl demonstrates a first application of frustrated composites in architectural scale.

Blurring boundaries between matter and structure, controlling macro - scale behaviour through micro and meso-scale material properties and structure, novel architectured fibre composites can be developed. Eliminating the moulds and learning from nature, we can imagine future resilient and sustainable architectural applications.

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Infinitesimal origami maps for one-degree-of-freedom metasurfaces

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Metasurfaces are composed of basic units with certain geometric features repeated in regular and/or irregular patterns to achieve certain design objectives. Metasurfaces include for instance origami, kirigami, tessellated structures and can be thought as 2D counterpart of metamaterials.The design and control of metasurfaces is a challenging task often addressed in the literature. In particular, metasurfaces may exhibit many deformation modes. As an example, it is well known that, given a folding pattern on a piece of paper, it is possible to obtain many different final shapes by varying the mountain-valley assignments.Here we investigate the possibility to design metasurfaces obtainable as the unique deformation of suitably tessellated plates.

To do so, we introduce the concept of infinitesimal origami maps, i.e., Lipschitz, piecewise affine maps whose gradient is almost everywhere a skew-symmetric tensor.We focus on a particular class of one-degree-of-freedom metasurfaces: those obtainable from infinitesimal origami maps of hexagonal, monohedral tessellations. Monohedral tessellated portions can be also joined together to obtain more complex shapes, which can be locally synclastic, anticlastic or monoclastic, and can even exhibit a certain finer-scale texture. Numerical and 3D printed experiments validate the theoretical results.The proposed metasurfaces can be controlled by a single actuator and may have applications in prosthetics, tissue engineering, biomechanical scaffolds, wearable devices, energy harvesting devices, tunable focus mirrors, and adaptive facades.

The talk is based on joint work with F. A. dos Santos, A. Favata, A. Micheletti, R. Paroni [1].

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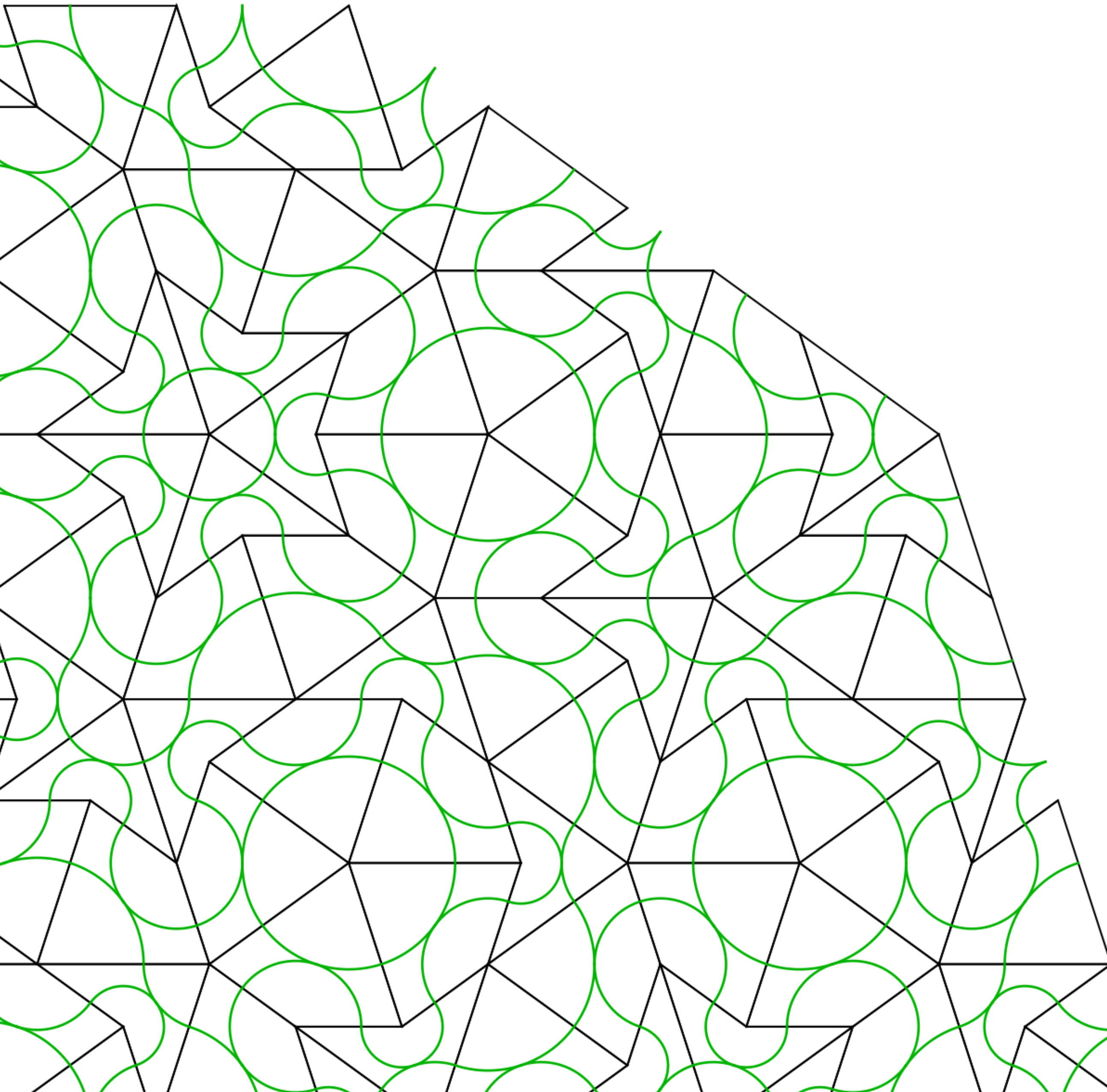
Session 4: Mechanics of active and frustrated systems



Truesdell Lecture

Some exceptional linkages, their continuum limits, and isometric deformations from helicoids to ruled Möbius bands

Eliot Fried

Mechanics and Materials Unit, Okinawa Institute of Science and Technology, Onna, Okinawa, Japan

Linkages, closed chains of interconnected links, possibly involving multiple loops, typically serve as mechanisms to induce motion or transmit force in mechanical systems. We introduce a family of single-loop linkages, each comprising $n \geq 7$ identical links connected by revolute hinges. Each such linkage in the family exhibits just one internal degree-of-freedom, manifested by an everting motion.

From the standpoint of the Chebyshev–Grübler–Kutzbach mobility criterion, any linkage of this family which has $n \geq 8$ links is underconstrained and, thus, deemed exceptional. In the asymptotic limit as $n \to \infty$, these linkages converge to a ruled Möbius band with three half twists and three-fold rotational symmetry. The rulings of this surface are aligned with the unit binormal of its midline, which is a geodesic and has uniform torsion. We demonstrate that this Möbius band can be obtained by a stable isometric deformation of a helicoid with a certain number of turns.

Moreover, helicoids with more turns lead to stable Möbius bands with more half twists. Among all stable Möbius bands with $k \geq 3$ half twists that are obtained by isometrically deforming helicoids, the one with k-fold symmetry has the least bending energy. While knotted Möbius bands can also be produced in this fashion, they turn out to be saddle points of the bending energy.

Finally, returning to the family of linkages first encountered in the talk, we present various consequences of relaxing the requirement that their links be identical, subject to a particular proportionality rule.

Round-table speakers

Extended kinematics of growing solids

Giuseppe Zurlo
University of Galway

Growth processes in solids lead to the accumulation of strain incompatibility. Classical continuum mechanical kinematics needs to be extended to capture the process of "delivery" of incompatibility.

To understand such processes, one needs to open up sub-continuum scales allowing one to reveal whether the delivered material is arranged compatibly or not inside the body.

The proposed micro-mechanical picture implies the introduction of a novel "tensorial displacement field", whose aim is to represent the mechanical frustration accompanying the assembly of a growing continuum. Based on a joint work with Lev Truskinovsky.

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Actin based motility unveiled: how chemical energy is converted into motion

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The chemo-mechanical motor of several physiological and pathological processes in biological systems is a polymerization process, which converts chemical energy into mechanical work. The chief component in this activity is actin, a multi-functional protein forming filament in the cell cytoskeleton. External impulses of a chemical or mechanical nature trigger a chemical reaction, which converts the monomeric form of actin, G-actin, into a polymerized branched- filamentous form, F-actin. Upon polymerization, the cross-linked network acts against the plasma membrane, a pathogenic bacterium or an endosome, pushing them forward and promoting directional motility.

At the leading edge of cells, actin is organized in an almost bidimensional dendritic array of branched filaments [1]. Branched actin filaments are generated beneath the plasma membrane by external signal responsive WASP-Arp2/3 machinery and kept functioning by a set of regulatory proteins. Those binding proteins control actin turnover and filament elongation, mediate the initiation of new filaments as branches on pre-existing filaments and promote (de)branching and (de)polymerization, thus regulating the mechanical response of moving cells.

In a recent publication [2], a thermodynamically consistent continuum - mechanics formulation was proposed, stemming from continuity equations that account for actin chemical kinetics. We have suggested that the volumetric expansion exerted after the phase change from monomeric to a cross-linked network of actin filament ultimately converts chemical energy into motion. In this note, the formulation in [2] will be extended and unpublished results presented for the first time. The main novelty is the application of Helmholtz free energies with no entropic contributions.

Numerical simulations of Listeria pathogens, with data taken from biological literature, show that the main features of actin-based motility are captured with remarkable accuracy. The model manifests itself in macroscopic descriptors of biochemical and biological details of the relevant processes, thereby resulting in sufficient generality to be appropriate for several biological systems, targeting cellular motility as a whole.

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Exploring bone remodeling: bridging generalized continuum mechanics and mechanobiology

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Bone remodeling is a dynamic and complex process through which bone tissue is constantly renewed and reshaped throughout our lives. It is a finely regulated process that involves the coordinated activity of specialized cells, namely bone-resorbing osteoclasts, bone-forming osteoblasts, and osteocytes.

The primary purpose of bone remodeling is to maintain the integrity and strength of our skeletal system. It allows for the removal of old or damaged bone tissue and the subsequent formation of new bone, ensuring that bones remain structurally sound, adapt to mechanical stresses, and participate in essential functions such as mineral homeostasis. The process of bone remodeling is tightly regulated by various biochemical and mechanical factors. Mechanobiology plays a significant role in bone remodeling, as mechanical forces and loading patterns influence the activity of bone cells, helping to shape bone structure in response to functional demands.

Our study introduces a tissue-scale model of bone remodeling that is built upon the theory of material remodeling, which belongs to the realm of generalized continuum mechanics. This framework is specifically designed to capture the adaptive behavior of living materials.

The focus is set on bone turnover, which refers to the biological process through which bone tissue continuously renews and adapts to its prevailing mechanical and biochemical environment. In our model, macroscopic bone turnover is the result of microscopic biological and biochemical processes, including the concerted activity of osteoclasts and osteoblasts, as well as the mineralization of the newly-formed bone matrix.

The proposed modeling approach provides a suitable framework to incorporate mechanobiology into a thermodynamically consistent mechanical framework. It also paves the way for a multiscale description of bone remodeling, integrating cell dynamics within a continuum model established at the tissue scale.

The discussion of preliminary numerical results aims to emphasize the key aspects of the model and to lay the foundation for future research.

Compressive stress and molecular deformation in the cell nucleus

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The mechanical impact of the nucleus on cellular function becomes evident during migration in 3-D environments. With its large volume and relative rigidity, governed by the nuclear envelope proteins and internal chromatin organization, the nucleus acts as physical barrier, particularly relevant to immune cells and invading cancer cells. Such cells must move through tissue pores and clefts, often smaller than the size of the nucleus, which induce substantial compressive and shearing stresses, and may also lead to the temporary rupture of the nuclear envelope.

Recent studies amply demonstrated that extreme nuclear deformations during confined migration, can lead to DNA damage and increased genomic instability, notably in cancer cells. Here we shed a first light on the molecular processes of stress transfer and relaxation, down the scale of the individual chromatin units, the nucleosomes. I will firstly outline our innovative experimental techniques, aimed at measuring nano-mechanical signatures of cancer cells, notably single-cell MEMS nanotweezers, which provide high sensitivity to examine different biophysical properties (size, stiffness, viscosity, etc.), and high-throughput, continuous-flow MEMS devices, oriented at clinical applications.

I will briefly discuss the theoretical coarse-graining of chromatin, going from the visco-elastic description to a discrete, self-avoiding polymer.

Then, I will show molecular-scale simulations of force-induced deformation of minimal chromatin elements, under ideally controlled conditions. External forces acting on multi-nucleosome particles transmit a mechanical stress, which is mainly translated as deformation energy stored in the elastic and plastic response of the DNA, which is wrapped around and links the nucleosome cores.

The ability of the double-stranded DNA helix to absorb and release this mechanical stress, most notably in the form of kinking, bending and twisting deformations, may constitute a framework to elicit or repress the interaction with remodeler proteins, by controlling their access to active chromatin domains. The concerted action of mechanical deformation and remodeler enzymes opens the way for a new paradigm, to understand the microscopic control of chromatin organization by mechanical forces, and the downstream effects on gene expression and transcription/repair factors activity.

Asymmetric elasticity of contact-based architected materials and associated wave dynamics

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Architected materials with internal contacts represent a relatively novel and yet not fully explored class of architected materials. Besides specific dissipative properties and path-dependent behavior, such materials can exhibit a distinctive property of elastic asymmetry.

In the first part of this work, we explore this asymmetry across different model architectures. In the second part, we examine wave propagation in media with pronounced elastic asymmetry. We use finite-element based homogenisation to investigate the elastic asymmetry and anisotropy of several model architectures with frictional and frictionless contact interfaces in infinitesimal and finite deformation regimes. We thereby construct their anisotropic and direction-dependent elasticity tensors. Dynamic behavior is scrutinized by studying wave dynamics within elastically asymmetric rods using finite difference methods [2].

This analysis sheds light on wave interactions and dispersion within these complex materials. Notably, we revisit the trajectories of specific points such as shocks, signotons, semi-signotons, and simple discontinuities, according to the classification by Maslov and Mosolov [1].

Our approach builds on their pioneering work on thermodynamically consistent descriptions of wave propagation in elastically asymmetric or "hetermodular" media. This research expands our understanding of architected materials with the pronounced elastic asymmetry induced by tuned internal contacts. This study opens new avenues for the design of novel architected materials for various engineering applications.

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Ultrasound characterization of multiphase architectured media: Insights into the mechanics of bio-mimicking systems

Nicolas Bochud[1], M. Gattin[1], G. Rosi[1], Q. Grossman[2], D. Ruffoni[2] and S. Naili[1]

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Tissue engineering scaffolds employed for bone tissue regeneration are multiphase architectured media, which usually consist of periodic arrays of soft inclusions (tissue, fluid, voids, or some combination thereof) embedded in a hard scaffold matrix.

The primary process at play during their insertion is the scaffold resorption, together with the gradual bone in-growth within the pores, which occurs at a length scale of a few hundred micrometers.

Their effective mechanical behavior at the tissue scale results from the combination of two main factors [1], namely (i) the properties of the constituent phases (soft or hard) and their respective volume fractions; and (ii) the presence of a structural organization (periodic microstructure), not to mention that both factors are time dependent (at the biological time scale).

As bulk phononic crystal analogs, scaffolds should therefore support elastic wave propagation in the megahertz (MHz) regime, whose frequency-dependent nature is expected to arise from a combination of internal resonances of the unit cells and wave interferences (Bragg scattering), which take place at frequencies where the effective wavelength is commensurate to twice the periodicity constant [2].

In this context, the rational design of multiphase architectured scaffolds that display an acoustic signature reflecting their microstructure could open the way towards the development of ultrasound characterization methods for the monitoring of their integration to the surrounding biological environment. To this end, it is necessary both to have a precise knowledge of the acoustic properties of the constituent materials and to accurately model the impact of the microstructure.

In this work, we discuss the capability of a multi-material 3D printing technology to design biomimicking micro-architectured media with programmable ultrasonic responses, with the aim of replicating such multiphase scaffolds formed by sub-millimeter unit cells in a controlled laboratory environment. First, the viscoelastic properties of the constituent phases are identified by characterizing homogeneous samples in the MHz regime, which exhibit dispersive losses that are described using a frequency power law model [3].

These properties are then used to feed models of the propagation of longitudinal ultrasound waves through time-evolving multiphase architectured media [4]. In particular, transmission and reflection spectra are computed for the viscoelastic, finite-size medium using a finite element method in the frequency domain.

This approach allows disentangling the relative contributions of viscoelasticity and periodicity on ultrasound signatures such as dispersion, attenuation, and bandgaps localization. Finally, the modeling outcomes are confronted with experiments conducted on 3D-printed samples, which exhibit a 2D periodicity at a length scale of a few hundreds of micrometers. Altogether, the obtained results shed light on the modeling characteristics to be considered when predicting the complex acoustic behavior of architectured media in the ultrasonic regime.

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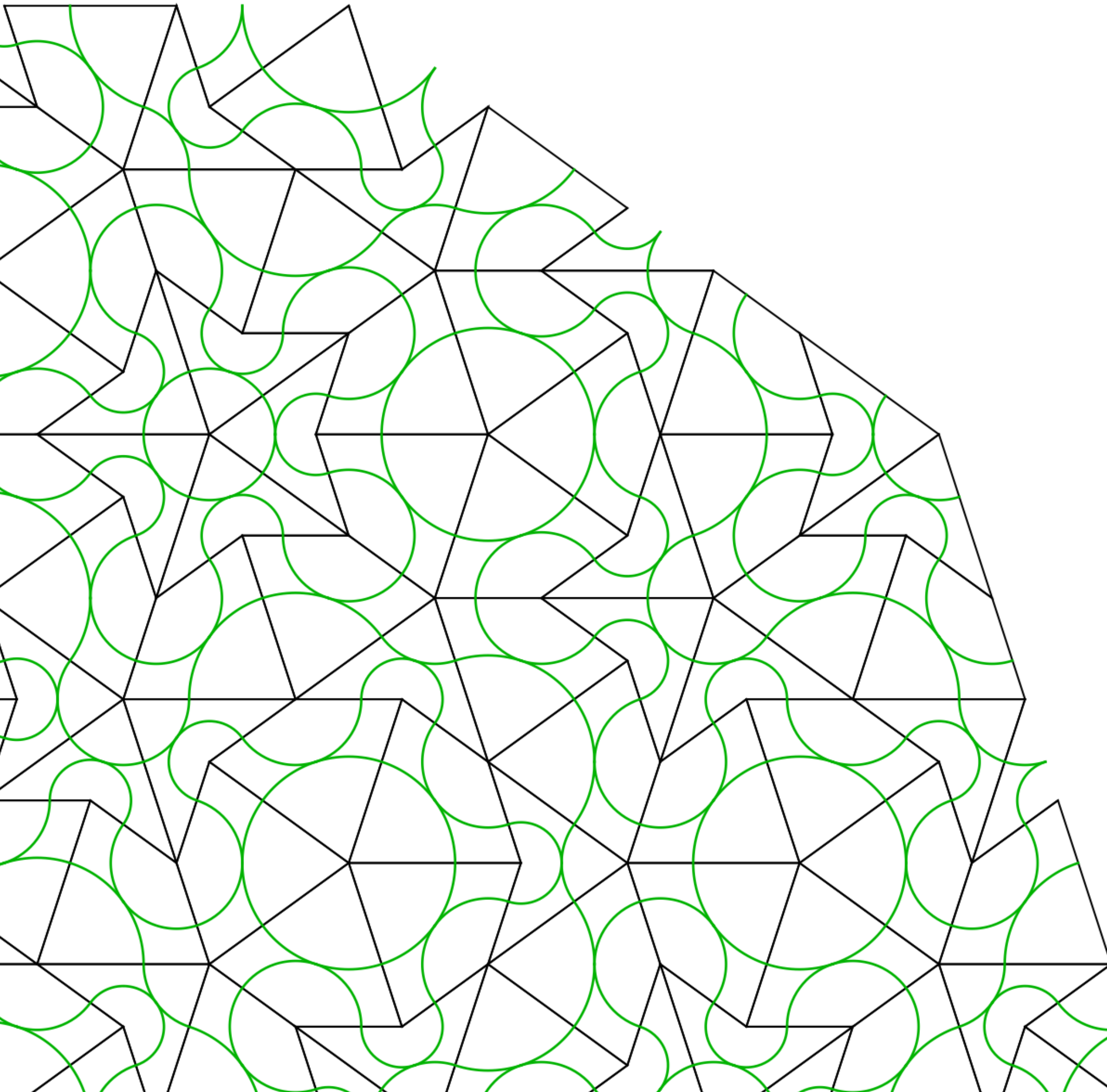
Session 5: Amorphous, glassy and granular systems



Keynote lecture

Mechanical instabilities enable spontaneous road building in the extracellular matrix and facilitate cancer metastasis

Phoebus Rosakis
Applied Mathematics, University of Crete

Just by squeezing the fibrous extracellular collagen surrounding them, contracting fibroblast cells spontaneously build roads that join distant cells or tumour clusters. These densified paths appear as if by magic in the extracellular matrix upon cell contraction.

We propose that these road patterns are actually the result of a densification phase transition, which is a macroscopic manifestation of a number of microscale mechanical instabilities inherent in a network of flexible fibers.

We test this hypothesis in multiple ways: (A) Microscopic modelling of individual fibers and their buckling, and of discrete fiber networks, (B) mesoscopic continuum modelling that sponatenously yields a multiwell stored energy starting from single fiber energy, (C) experiments with robotic cells that confirm this is a mechanical effect, (D) finite-element energy minimization computations that capture observed densification patterns and their complex microstructure.

This actually exhibits fine phase mixtures from nonconvex energy minimization, that are familiar from martensitic twinning in crystals. The biological role played by these densification patterns in tumour invasion cancer metastasis will be discussed.

Round-table speakers

Frame-Dependent stress tensors in dilute granular shearing flows

James T. Jenkins and Chao Zhang
Cornell University, Ithaca, USA

The stress tensor in dilute shearing flows of granular materials, in which the grains interact throughbinary, inelastic collisions, may be calculated through methods of kinetic theory. Examples of such flowsare those of sand, driven into collisions above a bed by a strong wind [1], and the orbital shearing flows of grains of ice in the rings of Saturn [2].

The stress in these flows is the product of the average mass density of the flow and the second momentof the velocity distribution function. The balance equation for this second moment may be obtained throughintegration from Boltzmann's equation. The resulting balance equation is not frame indifferent.

This is incontrast to the frame indifferent evolution equations for symmetric, second-rank tensors, such as thatintroduced by Hand [3], to describe anisotropic fluids.Because of the lack of frame indifference of the balance equation, the stress tensor in dilute, granular,collisional, shearing flows is also not frame indifferent, and depends on both the stretching and the spin ofthe flow. Müller [4] was the first to point out how frame indifference breaks down in the context of the kinetic theory. The dilute granular flow provides an example of itsbreakdown in a macroscopic, rather than a molecular, system.We indicate how the solution for the stress is obtained in dilute, steady, homogenous, rotationalshearing flows of inelastic, circular disks and show the results, as exemplified in the figure below.

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Continuum modelling of granular liquid crystals

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Granular flows composed of non-spherical particles are encountered in a number of applications, ranging from industry to Granular flows composed of non-spherical particles are encountered in a number of applications, ranging from industry to geophysics. Reliable mathematical models of flows of particles with complicated shape would permit, e.g., to increase the efficiency and decrease the energy required for handling and transporting granular materials in industrial apparati.

Discrete modelling of granular flows composed of spheres have been around for a long time and are now able to deal with a number of complicated effects such as aggregation, breakage, cohesion and poly-dispersity [1]. Continuum models that extend the seminal works on kinetic theory of granular gases to account for strong inelasticity, friction, velocity correlation, finite stiffness and presence of rate-independent components of the stresses can now satisfactorily reproduce the flows of rigid and soft spheres in a number of geometrical configurations [2-4]. In the last decade, discrete element simulations of shearing flows of true cylinders [5] and spherocylinders [6] have also been carried out. These simulations confirmed the experimental observation [7] that non-spherical particles, in which the ratio of major-to-minor axis is sufficiently far from one, develop a preferential alignment. This has a number of consequences on the constitutive relations to be adopted in continuum models. In particular, it means that at least the mean orientation angle should be treated as an additional state variable, while phrasing the associated evolution equation [8]. It has been shown that kinetic theory of granular gases is capable of predicting stresses and velocity fluctuations in homogeneous shearing flows of frictionless cylinders in a range of length-to-diameter (aspect) ratio and solid volume fraction [9], at least if preferential alignment is not dominant.

Here, we extend kinetic theory of granular gases to deal with particles that can manifest significant preferential orientation in response to shearing. This is done by incorporating the crucial role of particle velocity fluctuations in existing, continuum balance equations derived for liquid crystals.

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On the failure of quasi-periodic architectured materials

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The notion of quasi-periodic function goes back to [3] in 1893, but the first paper dealing withthe possible tiling of the plane with a finite set of tiles dates from [4]. Mathematicians have thenproposed various quasi-periodic tilings of the plane, the most famous being due to Penrose [7]and Amman and Beenker [1,2].

The subject still holds the attention of this community asillustrated by the recent work in [9]. After these achievements of mathematicians, the discoveryof the first quasicrystal in 1984 by D. Shechtman et al. [8], who received the 2011 Nobel Prizein Chemistry, closed the gap with reality. Subsequently, quasi-periodicity is a feature that has also been exploited in photonics and even in virology.

Recently, we have been trying to draw inspirationfrom the quasi-periodic tilings proposed by mathematicians to create architectured materials. By-creating architectures based on a few particular quasi-periodic patterns, we were able to start studyingtheir mechanical behaviour. According to our results, these materials have extraordinary properties, particularly with regard to cracking [6], but also with regard to elasticity [10] and vibration. However, we are currently unable to explain why these architectures have such properties.

Is it the symmetry class of the tiling? the shape of the tiles? the way of using the tiling to create apattern, by placing holes at the vertices of the tiles or in their centres?

In [5], crackingsimulations on hole arrays based on the same paving but using a different pattern gives different crackmorphology. We also show that their elastic properties at different observation scales show thepersistence of patterns of increasingly large sizes. This certainly comes from the ambiguity thatcharacterises their structure, which is both ordered and disordered. Observing the response ofquasi-materials at different scales appears to be essential and the secret of these materials probablycomes from the paths that the energy fluxes follow in their structure.

Presenting our recent observations and interpretations of the behaviour of quasi-periodic architectured materials is the mainpurpose of the proposed lecture.

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Nucleation and development of multiple cracks in thin composite fibers via the inverse-deformation approach

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We present results obtained in [1] concerning the nucleation and development of crack patterns in thincomposite fibers under tensile hard loading. A fiber comprises an elastic core and an outer layer of aweaker brittle material. In recent tensile experiments on such biological composites, multiple, equallyspaced cracks were observed to develop simultaneously on the outer layer [2].

We propose a simpleone-dimensional model to predict such phenomena using the inverse deformation approach to fracturefirst proposed in [3]. This method overcomes the problem of discontinuous deformations without usingadditional damage/phase-field variables and without pre-existing cracks. The main idea of this methodis to solve for the inverse deformation $h = f^{-1}$, which is piecewise smooth even when the originaldeformation f has discontinuities describing cracks.

We provide a weak formulation of the problemin the presence of the unilateral constraint that the inverse-strain is nonnegative, i.e., $h' \geq 0$. Thisenables a global bifurcation analysis that leads to existence results allowing for fractured solutions. Wecompute these solutions using FE discretization with continuation and active-set methods. The latter isemployed to incorporate the unilateral constraint during numerical computations. The route to failurein [3] of a homogeneous fiber is a single crack at one of the two ends. The effective stress drops to zero(identically) upon nucleation, after which the crack opens freely under hard loading. In contrast, thefractured outer layer with multiple cracks continues to interact with the elastic core of the compositefiber in our problem. Thus, crack development presents a delicate problem here, not arising in [3].

We present solutions exhibiting the spontaneous nucleation of cracks and their subsequent openingunder increased loading in Figure 1. An unstable pitchfork branch of inhomogeneous solutions bifurcatefrom the stress-strain curve at a critical load. This branch terminates at point C where multiple,simultaneous cracks nucleate on the outer layer of the composite. At this point, the unilateral constraint?activates? and a new stable, secondary solution branch emerges, along which the cracks open. Figure1(b) C, D shows configurations with finite-jump discontinuities marked in red. The crack faces maintainno elastic interaction between each other and precisely delineate the brittle material from empty spaces(or vacuum) in the deformed configuration

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